Name:

Math 10a
December 3, 2014
Quiz #10

1. (2 points) Let X be the number of heads in ten flips of a fair coin. What is $P(4 \leq X \leq 6)$?

$$\binom{10}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^6 + \binom{10}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^5 + \binom{10}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^4 = \frac{21}{32}.$$

2. (4 points) Let's model the number of free throw attempts before Shaquille O'Neal makes a free throw (where 1 attempt will mean he makes it the first time). The chance it takes him precisely k attempts is given by

$$p(1-p)^{k-1}.$$

   If in different games it takes him 3, 3, 1, 1, 2, 5, 2, 1, 1, and 3 attempts to make a free throw, what is the maximum likelihood estimate for p?

$$\mathcal{L}(p) = p(1-p)^{3-1}p(1-p)^{3-1}p(1-p)^{1-1}\cdots p(1-p)^{3-1} = p^{10}(1-p)^{12}$$

$$\ln \mathcal{L}(p) = 10\ln(p) + 12\ln(1-p) \xrightarrow{\frac{d}{dp}} \frac{10}{p} - \frac{12}{1-p} = 0 \Rightarrow p = \frac{5}{11}.$$

3. (4 points) You suspect a *ten-sided* die is weighted to land more often on 1 than on the other numbers. To test your suspicions, you roll the die 400 times and it lands on 1 fifty of those times.

   Using a significance level of $\alpha = .01$, do you have evidence to back up your suspicions?

   Assume the die is fair. If this is the case, then the number of times it lands on 1 during the 400 rolls is a binomial random variable: 400 trials, each with a 1/10 chance of success. It has a mean of 40 and a standard deviation of $\sqrt{400(1/10)(9/10)} = 6$. Since 400 is a large number of trials, the number of 1s is approximately normal, and hence the chance that one could get 50 or more 1s in those 400 trials is

$$P\left(Z \geq \frac{50-40}{6}\right) = P(Z \geq \frac{5}{3})$$

   which is clearly greater than .05. Hence getting 50 heads isn't so weird and, with a significance level of $\alpha = .01$ isn't enough to have evidence that the die is unfair.